$$a_7 = \left(\frac{f' \mathrm{d}x}{D'}\right) \cdot \left[1 + \frac{D'(1+C)}{4B}\right] - \frac{3}{2}y' \tag{13}$$

In these equations, D' denotes the hydraulic diameter at the entrance to the water channel, y' is the equivalent of y in that it is the ratio of the velocity component (in the direction of the main flow) of the mass of water being added or subtracted to the velocity of the main flow. The term C is a function of the change in the channel width in the flow direction and is defined as

$$C = \left[1 + \left(\frac{\mathrm{d}B}{\mathrm{d}x}\right)^2\right]^{1/2} \tag{14}$$

Equation (5) is the hydraulic equivalent of Eqs. (1) and (3), and a comparison of Eq. (5) with either Eq. (1) or (3) shows that the one-to-one correspondence between the A and B terms as demonstrated in Ref. 9 is now modified by the water friction through terms a_1 and a_4. A comparison of the remaining terms in Eq. (5) with those in Eqs. (1) and (3) shows that the combined effect of friction, internal drag, and changes in the gas composition, temperature, and mass flow rate can be simulated by a change in the water flow rate.

Equation (6) is the hydraulic equivalent of Eqs. (2) and (4), and a comparison of the former with either of the latter still shows a one-to-one correspondence between p and H^2. However, a similar relationship between M^2 and F^2 is altered through term a_5, which includes the effect of water friction and the velocity with which the water is added to or subtracted from the main channel flow.

An example of an application of the hydraulic equivalent is the hydraulic simulation of a confined, flowing, two-phase mixture injected with a combustible gas. Such a mixture will exhibit a mass change $\mathrm{d}\dot{w}$ due to the gas addition and changes in molecular weight $\mathrm{d}W$, specific heat ratio $\mathrm{d}\gamma$, and total energy $\mathrm{d}E$ due to combustion. Furthermore, the slower moving solid particles in the mixture will impose a drag $\mathrm{d}X$ on the gaseous flow and an uninsulated chamber wall could mean a heat loss $-\mathrm{d}Q$. Finally, this flowing, burning mixture might be used to deliver useful work $\mathrm{d}W_x$. In this case, Eqs. (3) and (4) express the combined effect of all these variations on the gaseous Mach number and static pressure. To find the equivalent water flow as expressed by these equations would require determining $\mathrm{d}p/p$ in Eq. (4) and then using Eqs. (5) and (6).

It is quite possible that sufficient information may not be available to simultaneously solve Eqs. (3) and (4) for $\mathrm{d}p/p$ in the preceding case. However, if $(\mathrm{d}p/p)$ for the flowing mixture is known from test data along with anticipated known values of $\mathrm{d}A/A$ and y, then the equivalent water flow can be determined. The procedure for determining this water flow is as follows:

1) Set $\mathrm{d}H^2/H^2 = \mathrm{d}p/p$ and $\mathrm{d}B/B = \mathrm{d}A/A$ in Eq. (6).

2) Calculate a_3, a_5, a_6, and a_7 assuming $f' = 0$ and $y' = y$.

3) Calculate $\mathrm{d}F^2/F^2$ from Eq. (6) and then the function $F = F(x)$ based upon the known boundary condition at the channel entrance $(x=0)$, $F = F_e = M_e$, as in Ref. 9.

4) With an average value of F over the simulation region, calculate the average water velocity and the value f' for a known water temperature.

5) Repeat steps 2-4 as often as necessary to obtain a final variation in F.

6) Calculate $\mathrm{d}\dot{w}/\dot{w}$ from Eq. (5) and then the function $\dot{w} = \dot{w}(x)$ based upon the known boundary condition at the channel entrance, $\dot{w} = \dot{w}_p$, where $\dot{w}_p = \dot{w}_p(M_e, B_e, H_g)$ denotes primary water flow rate as in Ref. 9.

In simulating the laser cavity flow of Ref. 9, the water flow was assumed frictionless. On this basis, the computed increase in water flow required to simulate the temperature increase was found to be 46.8%. It is interesting to note that when friction is considered and calculated using the expression of Ref. 11, the required increase in water flow was 36.4% assuming a water temperature of 20°C and normal injection $y=0$. This does not change the results of Ref. 9, since the resultant static pressure and not the temperature increase was simulated there. However, it does emphasize the importance of water friction in an analog study, particularly when water must be added to simulate a nonisentropic gas flow.

References

[1] Preiswerk, E., "Application of the Methods of Gas Dynamics to Water Flows with Free Surface," NACA TM Nos. 934 and 935, March 1940.

[2] Adams, D. M., "Application of the Hydraulic Analogy to Axisymmetric Nonideal Compressible Gas Systems," *Journal of Spacecraft and Rockets*, Vol. 4, March 1967, pp. 359-363.

[3] Mueller, T. J. and Oberkampf, W. L., "Hydraulic Analog for the Expansion-Deflection Nozzle," *AIAA Journal*, Vol. 5, June 1967, pp. 1200-1202.

[4] Laitone, E. V., "A Study of Transonic Gas Dynamics by the Hydraulic Analogy," *Journal of the Aeronautical Sciences*, Vol. 19, April 1952, pp. 265-272.

[5] Bryant, R. A. A., "Two-Dimensional Bow Shock Wave Detachment Distances," *Journal of the Royal Aeronautical Society*, Vol. 61, June 1957, pp. 424-426.

[6] Klein, E. J., "Interaction of a Shock Wave and a Wedge: An Application of the Hydraulic Analogy," *AIAA Journal*, Vol. 3, May 1965, pp. 801-808.

[7] Bryant, R. A. A., "The One-Dimensional and Two-Dimensional Gas Dynamics Analogies," *Australian Journal of Applied Science*, Vol. 7, Dec. 1956, pp. 296-313.

[8] Addy, A. L. and Mikkelsen, C. D., "An Investigation of Gas-Dynamic Flow Problems in Chemical Laser Systems," Gas-Dynamic Lab., University of Illinois, UILU-Eng-74-4009, Dec. 1974.

[9] Boraas, S., "Investigation of Jet Curtains for Chemical Laser Application," *AIAA Journal*, Vol. 16, March 1978, pp. 251-258.

[10] Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, Vol. 1, Ronald Press, New York, 1953, Chapt. 8.

[11] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, 1960, p. 503.

Heat and Mass Transfer in Close Proximity Impinging Two-Dimensional Laminar Jets

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Introduction

IMPINGING jets are of great practical interest in many industrial applications where they are commonly used as a means for attaining controlled heat and mass transfer rates on impingement surfaces. The study of jets impinging normally on a flat surface has been undertaken by many investigators. Gosman et al.[1] obtained solutions for the vorticity and temperature fields for the jet impinging at close proximity, but the transfer coefficients were not calculated. In the flow model of Gosman et al., the jet penetrated the solution domain with a fully developed Gaussian velocity profile of a free jet. In Scholtz and Trass,[2] the corresponding axisymmetric impingement flow was investigated in two steps; in the

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stagnation region, but outside the boundary layer, diffusion of vorticity was assumed to be negligible, and a solution was obtained for the vorticity field and freestream velocity distribution. In the second step, Scholtz and Trass sought a nonsimilar solution using the freestream velocity distribution obtained in the first step. The investigation by Silberman and Miyazaki[3] for an impinging jet with an initially uniform velocity profile was extended in a recent study by Sparrow and Lee[4] to the case of the impinging jet with initially nonuniform velocity profile. In both studies, it was assumed that the convection time based on the jet slot half-width, $d/\bar{u}$, is much smaller than the diffusion time d^2/ν. On this basis it was inferred that diffusion of vorticity was negligible.

The studies by Ero[5] and Gosman et al.[1] and the experiments of Donaldson and Snedeker[6] have shown that the jet impingement flow is characterized by two counterrotating fluid systems separated by a zero vorticity line. The magnitude of the vorticity increased on either side of the zero vorticity line. The component of velocity parallel to the deflecting wall was shown in these studies to attain a maximum on the zero vorticity line, thus the velocity distribution in the wall jet is similar to that found in a steady free convection flow on a vertical plane surface.

The aim of this study is to use the flow model of Refs. 1 and 5 to determine the transport coefficients in close proximity jet impingement. The result obtained will be compared with those obtained in Refs. 2-4. The influence of the initial velocity profile of the jet on the transport coefficients will be investigated. In view of the long computational times associated with this class of problems, only two velocity profiles at a jet-width Reynolds number of 1000 will be investigated.

Analysis

The Navier Stokes equation for momentum transport and the continuity equation for mass conservation can be manipulated to yield the vorticity transport and stream function equations:

$$\frac{\partial^2 \Omega}{\partial x^2} - Re\frac{\partial \psi}{\partial y}\frac{\partial \Omega}{\partial x} + \frac{\partial^2 \Omega}{\partial y^2} + Re\frac{\partial \psi}{\partial x}\frac{\partial \Omega}{\partial y} = 0 \quad (1)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\Omega(x,y) \quad (2)$$

where Ω and ψ are the vorticity and stream function, respectively, and Re the jet Reynolds number defined by

$$\Omega = \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}, \quad U = \frac{\partial \psi}{\partial y}, \quad V = -\frac{\partial \psi}{\partial x} \quad (3)$$

In these equations, the independent variables x, y are normalized by the slot half-width d, and the velocity components are nondimensionalized with the jet mean velocity. The Reynolds number is based on the jet mean velocity.

The energy and mass transport equations may similarly be written in the form

$$\frac{\partial^2 T}{\partial x^2} - RePr\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial y^2} + RePr\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = 0 \quad (4)$$

and

$$\frac{\partial^2 C}{\partial x^2} - ReSc\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial y^2} + ReSc\frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = 0 \quad (5)$$

where T and C are dimensionless temperature and concentration variables defined by

$$T = (\bar{T} - \bar{T}_w)/(\bar{T}_0 - T_w), \quad C = (\bar{C} - \bar{C}_w)/(\bar{C}_0 - \bar{C}_w) \quad (6)$$

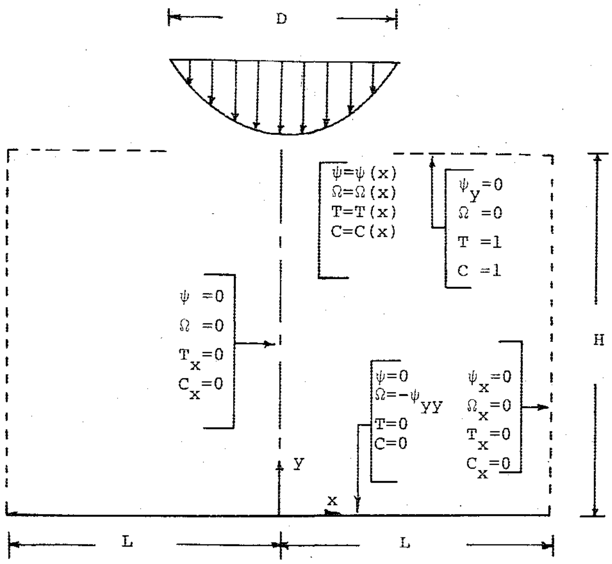

Fig. 1 Sketch of solution domain $H = 1$, $L = 1$.

where the overbars represent dimensional variables, and the subscript 0 represents mean variables at the jet entry into the solution domain.

The solution domain for Eqs. (1, 2,4, and 5) is shown in Fig. 1. The boundary conditions for these equations summarized in Fig. 1 are specified in the manner proposed by Refs. 1 and 7.

The three transport equations can be put in a compact form in terms of the generalized variable ϕ:

$$\alpha\frac{\partial^2 \phi}{\partial \xi^2} + 2Re_\phi \beta\frac{\partial \phi}{\partial \xi} + \gamma\frac{\partial^2 \phi}{\partial \eta^2} + 2Re_\phi \delta\frac{\partial \phi}{\partial \eta} = 0 \quad (7a)$$

in which ϕ may be replaced by Ω, T, or C, and Re_ϕ succesively represents Re, $RePr$, and $ReSc$.

$$\alpha = \left(\frac{\partial \xi}{\partial x}\right)^2, \quad \beta = \frac{1}{2}\left(\frac{1}{Re_\phi}\frac{\partial^2 \xi}{\partial x^2} - \frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial y}\frac{\partial \psi}{\partial \eta}\right) \quad (7b)$$

$$\gamma = \left(\frac{\partial \eta}{\partial y}\right)^2, \quad \delta = \frac{1}{2}\left(\frac{1}{Re}\frac{\partial^2 \eta}{\partial y^2} + \frac{\partial \xi}{\partial x}\frac{\partial \eta}{\partial y}\frac{\partial \psi}{\partial \xi}\right) \quad (7c)$$

The new variables ξ, η are suitably stretched functions of x and y, respectively, and may be represented by the general form $\xi = \xi(x)$, and $\eta = \eta(y)$.

Method of Solution

An augmented central difference algorithm[5,8] was used in discretizing the generalized governing equation. In this differencing scheme, the first-order derivative is approximated by the usual central difference plus a correction term which depends on a higher-order derivative. The scheme has second-order accuracy and yields algebraic equations that are diagonally dominant for all values of mean Reynolds number. Details of the development of the augmented central difference scheme can be found in Refs. 5 and 8. The solution of the discretized equations was obtained iteratively, using the Liebman method, with optimal relaxation factor found by trial and error.

Result and Discussion

The vorticity and stream function results of this study showed an enhanced entrainment zone, a result which is in good agreement with Gosman et al.[1] and Bower et al.[8] The flow in the stagnation region and in the wall jet is influenced primarily by the main jet flow. This observation is consistent

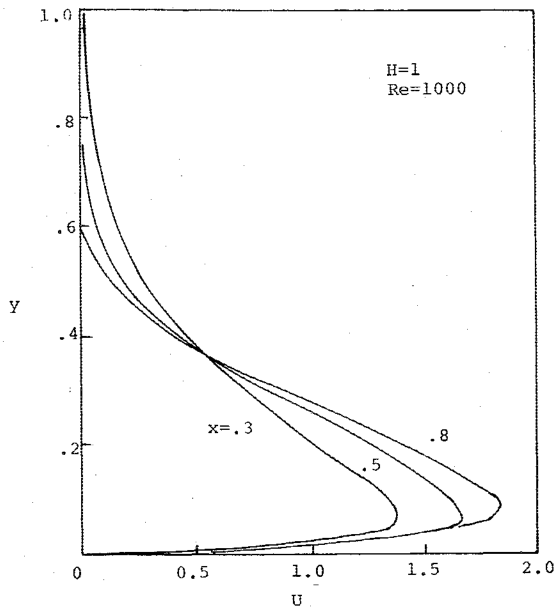

Fig. 2 Velocity distribution in the wall jet.

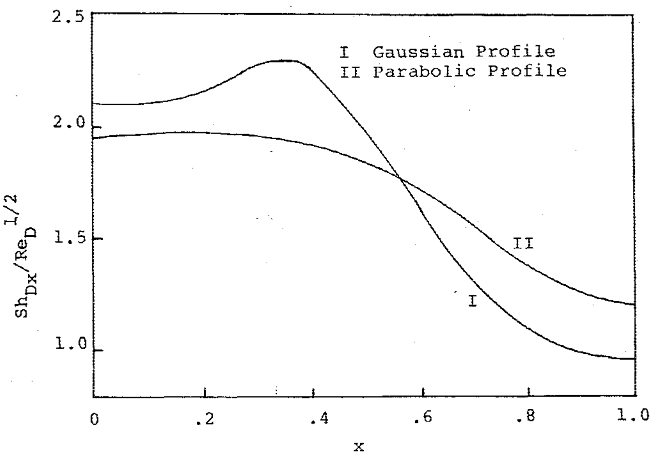

Fig. 3 Variation of local Sherwood number ($Sc = 2.45$).

with the model of Sparrow[4] which assumed negligible diffusion of vorticity in the direction normal to the axis of the primary jet flow. The velocity profiles in the wall jet are shown in Fig. 2, the distance from the deflecting wall to the point of maximum velocity increases with distance from the stagnation point. However, this distance from the wall does not correspond to a boundary-layer thickness as assumed by the Scholtz model.[2] The U-velocity profile was found to be invariant for $x \geq 0.25$; a freestream velocity in the sense discussed by Scholtz and Trass was not confirmed. This observation was independent of the initial jet velocity profile examined in this study. The profiles in Fig. 2 agree well with those obtained experimentally by Donaldson et al.[6]

The results for the heat and mass transfer computation are shown in Figs. 3 and 4, and the corresponding initial velocity profiles are shown inset in Fig. 4. In the mass transfer study, the property of Naphthalene is assumed, $Sc = 2.45$. The parabolic initial profile predicts Sherwood number within 2% of the experimental data of Sparrow.[4] The Gaussian profile overpredicted data by 9%. The Nusselt number obtained in this study is in good agreement with the values obtained for the stagnation region of a cylinder in crossflow.[9]

Conclusions

The variation of heat and mass transfer parameters along a deflecting surface is controlled by the initial velocity profile in

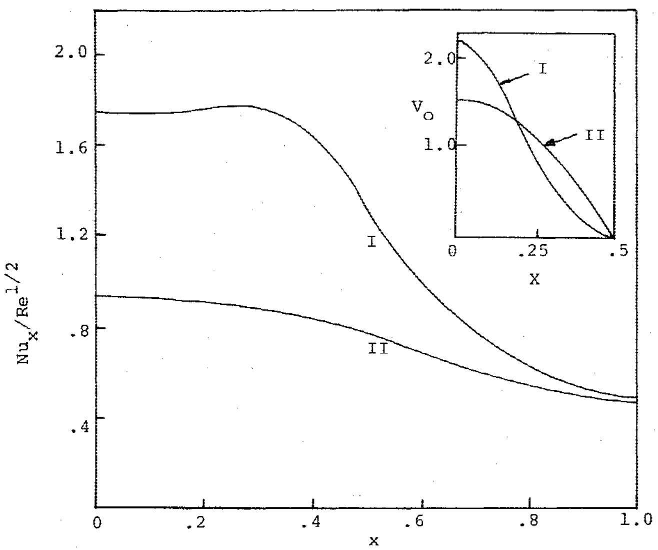

Fig. 4 Local heat-transfer coefficient ($Pr = 0.72$, $H = 1$).

a close-proximity impinging jet. The difference between the transfer parameters obtained for the Gaussian and parabolic velocity profiles may be explained by the quantity of high-velocity fluid delivered to the deflecting wall. For the same mean flow velocity based on jet width, the Gaussian profile attains a higher centerline velocity. The effect of the entrainment of ambient fluid did not seem to penetrate the wall jet.

References

[1] Gosman, A. D., Pun, W. M., Runchal, A. K., Spalding, D. B., and Wolfshtein, M., *Heat and Mass Transfer in Recirculating Flows*, Academic Press, New York, 1969.

[2] Scholtz, M. T. and Trass, O., "Mass Transfer in a Non-Uniform Impinging Jet, Parts I and II," *AIChE Journal*, Vol. 16, Jan. 1970.

[3] Silberman, E. and Miyazaki, H., "Flow and Heat Transfer on a Flat Plate Normal to a Two-Dimensional Laminar Jet Issuing from a Nozzle of Finite Height," *International Journal of Heat and Mass Transfer*, Vol. 15, Nov. 1972.

[4] Sparrow, E. N., and Lee, L., "Analysis of Flow Field and Impingement Heat/Mass Transfer Due to a Nonuniform Slot Jet," *Journal of Heat Transfer, Transactions of ASME, Ser. C.*, Vol. 97, May 1975.

[5] Ero, M. I. O., "Vertical Impingement of Laminar and Turbulent Axisymmetric Jets," McDonnell Douglas Research Lab., Rept. No. MDC Q0600, Oct. 1976.

[6] Donaldson, D. P. and Snedeker, S., "A STudy of Jet Impingement, Part I, Mean Properties of Free Impinging Jets," *Journal of Fluid Mechanics*, Vol. 45, Pt. 2, Jan. 1971.

[7] Roache, P. J., *Computational Fluid Dynamics*, Hermosa Publishers, Albuquerque, N. Mex., 1972.

[8] Bower, W. W. and Kotansky, D. R., "A Navier-Stokes Analysis of the Two-Dimensional Ground Effects Problem," McDonnell Douglas Research Lab. Rept. No. MDRL 76-20, July 1976.

[9] Eckert, E. R. G. and Drake, R. M., *Analysis of Heat and Mass Transfer*, McGraw-Hill, New York, 1972.

Notes on the Transonic Indicial Method

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Introduction

RECENTLY, Ballhuas and Goorjian[1] presented a method of calculating flutter derivatives for two-dimensional

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